College Algebra: Problem Set 12 Exponential and Logarithmic functions, Properties of exponents and logarithms, Solving exponential and logarithmic equations

Determine the domain and range of the following functions and sketch.

1. $f(x) = 3^x$
2. $y = \log_2 x$
3. $g(x) = \log_4(x-1)$
4. $f(x) = e^x - 2$

Express each of the following as a single logarithm.

5. $\ln(3x+1) + 2\ln x, \quad x > 0$
6. $\log x + 2\log y - \frac{1}{2}\log z, \quad x, y, z > 0$
7. $\frac{1}{3}\log w + 3\log x - 5\log y, \quad w, x, y > 0$

Evaluate, if possible.

8. $\log_5 125$
9. $\log_7 7$
10. $\log_8 1$
11. $\log(-14.9)$

Simplify.

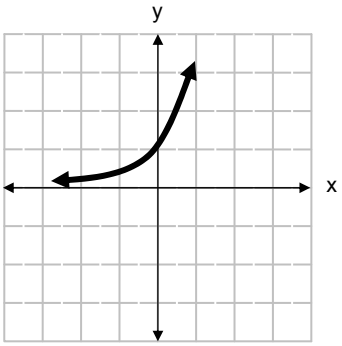
12. $2\log_6 6^{3x+1}$
13. $10^{\log(x+6)}$
14. $\ln e^x$

Solve the following equations.

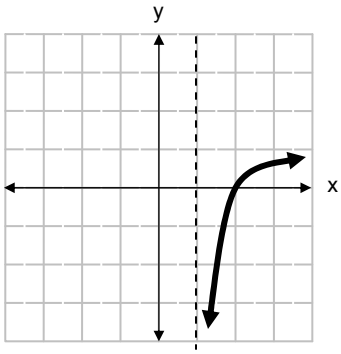
15. $4^{3x} = 8^{x+5}$
16. $6^{3x+5} = 55$
17. $4e^{-0.7x} = 5$
18. $5\ln x = 12$
19. $\log_8 x = \frac{1}{3}$
20. $\log x + \log(2x+5) = \log 7$
21. $\log x = 2 - \log(x-15)$
22. $3xe^{-x} - 12e^{-x} = 0$

Solutions to Odd Problems

1. D: $(-\infty, \infty)$ R: $(0, \infty)$



3. D: $(1, \infty)$ R: $(-\infty, \infty)$



5.

$$\ln(3x+1)+2\ln x, \quad x>0$$
$$\ln(3x+1)+\ln x^2$$
$$\ln[x^2(3x+1)]$$

7.

$$\frac{1}{3}\log w+3\log x-5\log y, \quad w,x,y>0$$
$$\log(w^{1/3}x^3)-\log y^5$$
$$\log\frac{w^{1/3}x^3}{y^5}$$

9. 1

11. undefined

13. x + 6

15.

$$4^{3x}=8^{x+5}$$
$$(2^2)^{3x}=(2^3)^{x+5}$$
$$2^{6x}=2^{3x+15}$$
$$6x=3x+15$$
$$3x=15$$
$$x=5$$

17.

$$4e^{-0.7x}=5$$
$$e^{-0.7x}=\frac{5}{4}$$
$$-0.7x=\ln\frac{5}{4}$$
$$x=\frac{\ln\frac{5}{4}}{-0.7}=\frac{10}{7}\ln\frac{4}{5}$$

19.

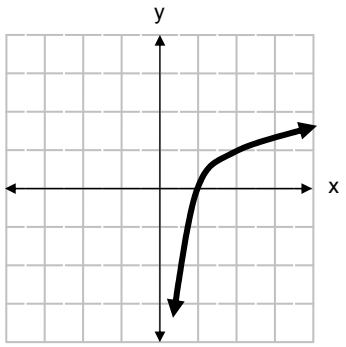
$$\log_8 x=\frac{1}{3}$$
$$8^{1/3}=x$$
$$x=2$$

21.

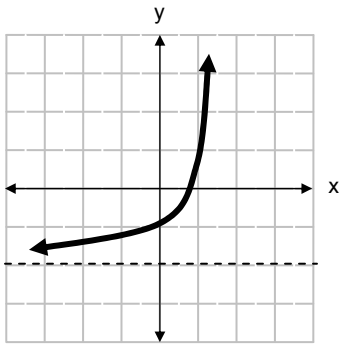
$$\log x=2-\log(x-15)$$
$$\log x+\log(x-15)=2$$
$$\log[x(x-15)]=2$$
$$10^2=x(x-15)$$
$$0=x^2-15x-100$$
$$x=20 \quad or \quad x=-5\ (does\ not\ check)$$
$$x=20$$

<u>Answers to Even Problems</u>

2. D: $(0,\infty)$ R: $(-\infty,\infty)$



4. D: $(-\infty,\infty)$ R: $(-2,\infty)$



6. $\log\frac{xy^2}{\sqrt{z}}$

8. 3

10. 0

12. $2(3x+1)$

14. x

16. $\frac{1}{3}(-5+\log_6 55)$

18. $e^{12/5}$

20. x = 1

22. x = 4