College Algebra: Problem Set 10 Distance and midpoint formulas, x and y intercepts, linear and quadratic equations in 2 variables, symmetry, translations, circles

Given the points P(2, 3) and Q(-4, 7), find the following.

1. The distance between P and Q.
2. The midpoint of segment $\overline{PQ}$.

Find the x and y intercepts of the following.

3. $5x + 2y = 8$
4. $y = 2x^2 - 3x - 2$

5. Find the slope of the line through the points (2, 3) and (-4, 7).

Write an equation for the following.

6. A line passing through (-2, 5) and (3, 9).
7. A vertical line passing through (1/2, -6)

8. A horizontal line passing through (3, -1).

9. A line parallel to $5x - 2y = 8$ and passing through (-2, 3).

10. A line perpendicular to $y = -\frac{2}{3}x + 5$ and passing through (4, 1).

Sketch a graph of the following.

11. $5x + 2y = 8$
12. $x = -3$

State whether each parabola opens upward or downward. Find the vertex and intercepts. Sketch.

13. $y = -2x^2 - 3x + 7$
14. $y = 3x^2 + 2x - 4$

Test each of the following for symmetry to the x-axis, y-axis, and origin.

15. $y = 5 + x^4$
16. $y = 2x^4 - 3x^2$
17. $y = x^3 + 2x$
18. $y^2 - 3x = 4$

Describe how the graph of $y = |x|$ can be transformed to obtain the graphs of the following equations.

19. $y = |x + 1| - 3$
20. $y = -|x - 2|$

Describe how the graph of $y = \sqrt{x}$ can be transformed to obtain the graphs of the following equations.

21. $y = -\sqrt{x - 3}$
22. $y = \sqrt{x + 2} + 1$

Find the center and radius of the circles with the given equations.

23. $(x - 2)^2 + (y + 1)^2 = 9$
24. $x^2 + y^2 = 6$

Solutions to Odd Problems.

1.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d = \sqrt{(-4-2)^2 + (7-3)^2}$$
$$d = \sqrt{52} = 2\sqrt{13}$$

3.

$$x - \text{int} : y = 0$$
$$5x + 2(0) = 8$$
$$x = \frac{8}{5} \quad \left(\frac{8}{5}, 0\right)$$
$$y - \text{int} : x = 0$$
$$5(0) + 2y = 8$$
$$y = 4 \quad (0,4)$$

5.

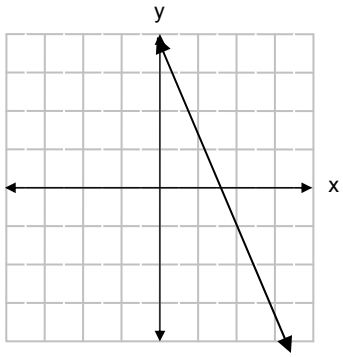
$$m = \frac{y_2 - y_1}{x_2 - x_1}$$
$$m = \frac{7-3}{-4-2} = \frac{-2}{3}$$

7. x = 1/2

9.

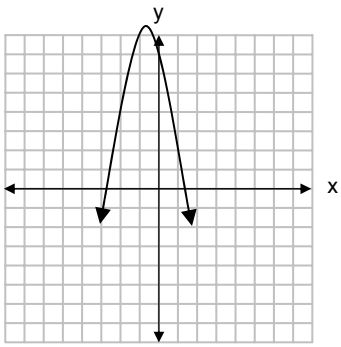
$$5x - 2y = 8$$
$$2y = 5x - 8$$
$$y = \tfrac{5}{2}x - 4$$
$$m = \frac{5}{2}$$
$$y - y_1 = m(x - x_1)$$
$$y - 3 = \tfrac{5}{2}(x+2)$$
$$y = \tfrac{5}{2}x + 8$$

11.



13.

$$y = ax^2 + bx + c$$
$$a < 0, \quad \textit{opens downward}$$
$$\textit{Vertex} : x = \frac{-b}{2a} = \frac{-(-3)}{2(-2)} = \frac{-3}{4}$$
$$y = -2(\tfrac{-3}{4})^2 - 3(\tfrac{-3}{4}) + 7 = 8\tfrac{1}{8}$$
$$\textit{vertex} : \left(\tfrac{-3}{4}, 8\tfrac{1}{8}\right)$$
$$x - \text{int} : \left(\frac{3 \pm \sqrt{65}}{-4}, 0\right)$$
$$y - \text{int} : (0,7)$$



15. x-axis: $(-y) = 5 + x^4$ no
y-axis: $y = 5 + (-x)^4$ yes
origin: $(-y) = 5 + (-x)^4$ no

17. x-axis: $(-y) = x^3 + 2x$ no
y-axis: $y = (-x)^3 + 2(-x)$ no
origin: $(-y) = (-x)^3 + 2(-x)$
$y = x^3 + 2x$ yes

19. Shift left one unit, down 3 units

21. Reflect across x-axis, shift right 3 units

23. C: (2, -1) r = 3

Answers to Even Problems

2. M: (-1, 5)

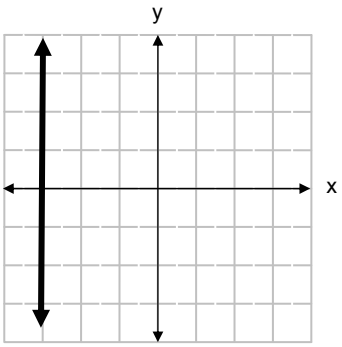
4. (0,-2), (2, 0), (-1/2, 0)

6. $y = \frac{4}{5}x + \frac{33}{5}$ *or* $4x - 5y = -33$

8. y = -1

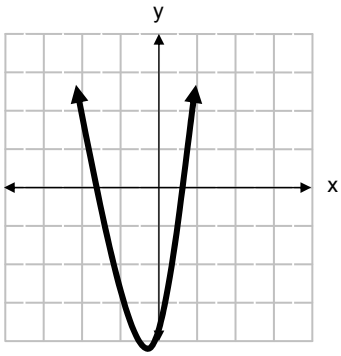
10. $y = \frac{3}{2}x - 5$ *or* $3x - 2y = 10$

12.



14. opens upward, V: $\left(\frac{-1}{3}, -4\frac{1}{3}\right)$,

intercepts: $(0,-4), \left(\frac{-1 \pm \sqrt{13}}{3}, 0\right)$



16. y-axis symmetry

18. x-axis symmetry

20. Reflect across the x-axis, shift right 2 units

22. Shift left 2 units, up 1 unit

24. C: (0, 0) r = $\sqrt{6}$